

Solve the following algebraically

① Given $f(x) = 3x^2 - x$, find

(a) the slope of the curve at any point $x=a$.

(b) When will the slope be -16 ?

② $\lim_{x \rightarrow 4} \frac{2x-8}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$$\lim_{x \rightarrow 4} \frac{2(x-4)(\sqrt{x}+2)}{(x-4)} \Rightarrow ,$$

$$\lim_{x \rightarrow 4} 2(\sqrt{x}+2) \Rightarrow 2(\sqrt{4}+2) = \boxed{8}$$

③ $\lim_{x \rightarrow 0} \frac{\sin 2x}{4x}$

$$\lim_{x \rightarrow 0} \left(\frac{1}{2} \cdot \frac{\sin 2x}{2x} \right)$$

$$\lim_{x \rightarrow 0} \frac{1}{2} \cdot \lim_{x \rightarrow 0} \frac{\sin 2x}{2x}$$

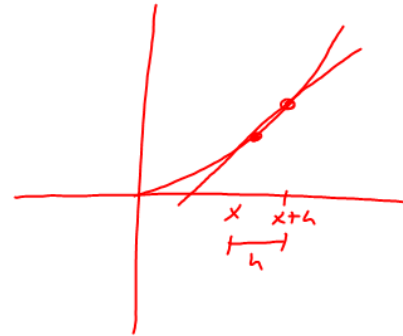
$$\frac{1}{2} \cdot 1 = \boxed{\frac{1}{2}}$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$

$$f(x) = 3x^2 - x$$

$$f(x+h) = 3(x+h)^2 - (x+h)$$

$$\frac{3(x+h)^2 - (x+h) - (3x^2 - x)}{h}$$



$$\lim_{h \rightarrow 0} \frac{\cancel{3x^2} + 6x\cancel{h} + 3\cancel{h^2} - \cancel{x} - \cancel{h} - \cancel{3x^2} + \cancel{x}}{h} = \lim_{h \rightarrow 0} 6x + 3h - 1$$

$$= 6x - 1$$

$$= 6a - 1$$

$$m = -16$$

$$-16 = 6a - 1$$

$$-15 = 6a$$

$$a = -2.5$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \text{derivative} = \underline{f'(x)}, \underline{y'}$$

f prime of x derivative of y
y prime

$$\underline{D_x[y]} \quad \underline{\frac{d}{dx} f(x)}$$

Derivative of
y with
respect to x

$$\underline{\frac{dy}{dx}} \text{ "d" "y" "d" "x"}$$

$$\underline{\frac{d}{dx} f} \text{ "d" "d" "x" of f}$$

$$\underline{\frac{df}{dx}}$$

Derivative:

The derivative of the function f with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}, \text{ provided the limit exists}$$

Derivative at a point: (alt. def.)

The derivative of the function f at the point $x=a$

is the limit
$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}, \text{ provided the limit exists}$$

Find the derivative of $f(x) = \sqrt{x}$ using both methods

$$\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{\cancel{x+h} - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} \Rightarrow \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \left(\frac{1}{2\sqrt{x}} \right), \frac{\sqrt{x}}{2x}, (2\sqrt{x})^{-1}$$

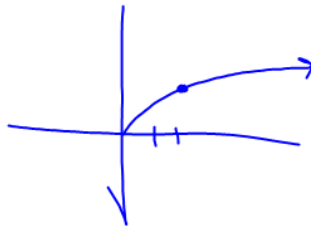
$$\lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} = \frac{\cancel{x} - \cancel{a}}{(x-a)(\sqrt{x} + \sqrt{a})} = \frac{1}{\sqrt{x} + \sqrt{a}}$$

$$\lim_{x \rightarrow a} \frac{1}{\sqrt{x} + \sqrt{a}} = \left(\frac{1}{2\sqrt{a}} \right) = f'(a)$$

$$f(x) = \sqrt{x}$$

m at 2

$$\frac{1}{2\sqrt{2}} = m$$



Sect. 3.1 # Quick Review #7,9,10

P.S. # 1-4,5,7,9,13-19,21,23,25,26,28